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DOI: 10.53704/fujnas.v6i2.150

A publication of College of Natural and Applied Sciences, Fountain University, Osogbo, Nigeria.

Journal homepage: www.fountainjournals.com

ISSN:2354-337X(Online),2350-1863(Print)

The Dynamic of Heterosexual Transmission of HIV/AIDS Model with Treatment

¹Bashiru K. A, ¹Ojurongbe T.A and ²Lawal Muhammed

¹Osun State University, Department of Mathematical Sciences P.M.B 4494 Osogbo, Nigeria. ²Fountain University, Department of Mathematical and Computer Sciences, Osogbo, Nigeria.

Abstract

A Mathematical Model of HIV/AIDS with Heterosexual transmission in the presence of treatment was examine in this paper, it ascertained the impact of treated individuals on the transmission dynamics of HIV/AIDS. Equilibrium points of the model system were found, stability analysis and numerical simulation were carried out, it was discovered that HIV/AIDS can die out with test of time as R_0 < 1. It was observed that the model had a disease free equilibrium which was asymptotically stable for R_0 < 1 and unstable for R_0 > 1. Graphical representations of the numerical analysis showing the effect of treatment on the model were also presented.

Keywords: HIV/AIDS, Heterosexual Transmission, ARVs, Compartmental model, Disease Free and Endemic equilibrium, Treatment, and Basic Reproductive Number.

Introduction

The human immunodeficiency virus (HIV) epidemic has spawned a scientific effort unprecedented in the history of infectious disease research. HIV/AIDS transmission dynamics study has been of great interest and the effort has merged aspect of clinical research, basic molecular biology, epidemiology and mathematical modeling in ways that have not been seen before.

The epidemic in Nigeria has extended beyond the commonly classified as high - risk groups and is now common in the general population. With the adult prevalence rate at 6.9% in 2003, the nation was said to be at threshold of an exponential growth of the epidemic (Bashiru, 2005).

By all indications the HIV/AIDS epidemic has contined to grow largely through Heterosexual unprotected sexual exposures (Bashiru et al 2009) Sexual intercourse is the principal mode of transmission, accounting for an estimated 75 percent of infections globally, of which 75 percent involve heterosexual intercourse and 25 percent sexual relations between men (Epstein 2009). In Third World countries, sexual transmission accounts for even higher proportions of infections. In Africa, Asia, and the Caribbean, infections are over - whelmingly heterosexual, with an estimated less than

*Corresponding author: +2348034997776 Email address: kehindebashiru@uniosun.edu.ng one percent through homosexual sex (Yanan et al 2013).

Number of studies involving treatment of HIV/AIDS have shown that when HIV/AIDS infected individuals are treated with antiretroviral drug and the number of babies born infected is significantly reduced, other studies[Gortmaker et al(2001) and shaffer et al(1999)] involving treatment of HIV infected individuals prove further that effective treatment increases the life expectancy of people infected.

In this study, we modified the model proposed by Bashiru et al (2009) who considered Mother - to - Child and Heterosexual modes of transmission of HIV/AIDS without treatment. Since antiretroviral drugs capable of prolonging life expectancy were unknown or not generally available, there is need to formulate a model with treatment to measure the impact of health policies aimed at controlling such epidemics. Therefore, we extended the model proposed by Bashiru et al (2009) with the introduction of treatment compartment.

Model Equations

We intend to propose a simple HIV/AIDS model with treatment class for Heterosexual mode of transmission. In this model, the sexually mature population is divided into four compartments: the Susceptibles, the Infectives (also assumed to be infectious), the Treatment and the AIDS population whose numbers are denoted by S,I,T and A. The number of total population is denoted by N(t), at any time t. In this model, we assumed that the susceptible become HIV Infected via sexual contacts with Infectives. We assumed that proportion of children that survive through age 15 and become sexually active join the susceptible class at the rate of pq. We also assumed that some of the Infectives move to join the treated class with the rate σ_2 and then proceed to AIDS class with rate k to develop full blown AIDS, while others with serious infection directly join the AIDS class with a rate σ_1 .

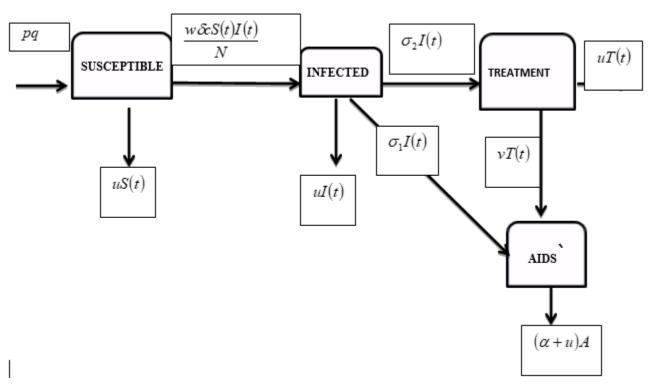


Fig.2.1: Proposed schematic diagram for the spread of HIV/AIDS.

With the above considerations and assumptions, the spread of the disease is assumed to be governed by the following system of nonlinear ordinary differential equations:

$$\frac{dS}{dt} = pq - \frac{w\delta cSI}{N} - uS$$

$$\frac{dI}{dt} = \frac{w \, \delta c SI}{N} - uI - \sigma_2 I - \sigma_1 I$$

$$\frac{dT}{dt} = \sigma_2 I - uT - vT \qquad ...1$$

$$\frac{dA}{dt} = \sigma_1 I + vT - (\alpha + u)A$$

Where:

 $\ensuremath{\text{c}}$ - Average number of sexual partners per unit time.

lpha - Disease - induced death rate due to AIDS.

 $\sigma_{\rm 2}\text{-}$ Rate of movement from infectious class to treatment class.

 $\sigma_{\scriptscriptstyle 1}$ - Rate of movement from infectious class to AIDS class.

u - Natural mortality rate.

v - Rate at which Treatment class moves to AIDS group.

 \boldsymbol{k} - Rate at which treated group becomes full blown AIDS.

 $\it w\,$ - Sexual contact rate between a sexually mature S person and I person.

 $\,p\,$ -The proportion of children that survive after age 15 years.

q - Infant Survival rate.

S(t) - Number of Susceptible individuals at time t.,

I(t) - Number of Infected individuals at time t .

T(t) - Number of individuals under ARV Treatment at time t .

A(t) - Number of individual with full - blown AIDS at time t .

Positivity and Boundedness of Solutions.

In this section, the positivity and boundedness of solution of equation 1 to 4 was established as the model described the evolution of population, hence the population should always remain non - negative and bounded, and this implies the global existence of

the solutions. We assumed that the initial data for equation (1) satisfies

$$S^{0} \ge 0, I^{0} \ge 0, T^{0}, A^{0} \ge 0$$
 ...2

Existence and Uniqueness of Solution for the Model.

For the model to predict the future of the system from its current state at time t_0 , the initial value problem (IVP)

$$x' = f(t, x),$$
 $x(t_0) = x_0$...3

Must have a solution that exists and also unique.

In this subsection, the conditions for the existence and uniqueness of solution for the model will be established.

let

$$f_1(t, x) = pq - \frac{w \delta cSI}{N} - uS$$

$$S(t_0) = S_0$$

$$f_2(t, x) = \frac{w \delta cSI}{N} - uI - \sigma_2 I - \sigma_1 I$$

$$I(t_0) = I_0$$

$$f_3(t,x) = \sigma_2 I - uT - vT, \qquad T(t_0) = T_0 \qquad \dots 4$$

$$f_4(t,x) = \sigma_1 I + \nu T - (\alpha + u)A \qquad \qquad A(t_0) = A_0$$

So that

$$x' = f(t, x) = f(x),$$

Theorem

Let D' denote the region

$$|t-t_0| \le a, ||x-x_0|| \le b, \ x = (x_1, x_2, ..., x_n), \ x_0 = (x_{10}, x_{20}, ..., x_{n0})$$
 ...5

And suppose that f(t,x) satisfies the Lipchitz condition

$$||f(t,x_1)-f(t,x_2)|| \le k||x_1-x_2||$$
 ...6

Whenever the pair (t,x_1) and (t,x_2) belong to D, where k is a positive constant. Then , there exist a constant $\delta>0$ such that there exist a unique continuous vector solution x(t) of the system (2) in the interval $|t-t_0| \leq \delta$ (Derrick and Grossman(1976))

It is important to note that the condition (6) is satisfied by requirement that $\frac{\partial f_i}{\partial x_i}$, i, j = 1, 2, ... n be

continuous and bounded in D

Lemma. 1. (see Derrick and Grossman(1976)), if f(t,x)continuous partial $\frac{\partial f_i}{\partial x_i}$, i, j = 1, 2, ... non a bounded closed convex

domain R, then it satisfies a Lipchitz condition in R

Proof.

Let,
$$\frac{dS}{dt} = f_1(S, I, T, A) = pq - \frac{w\delta cSI}{N} - uS$$

$$\frac{dI}{dt} = f_2(S, I, T, A, t) = \frac{w\delta cSI}{N} - uI - \sigma_2 I - \sigma_1 I$$
...7

$$\frac{dT}{dt} = f_3(S, I, T, A, t) = \sigma_2 I - uT - vT$$

$$\frac{dA}{dt} = f_2(S, I, T, A, t) = \sigma_1 I + \nu T - (\alpha + u)A$$

It suffices to show that $\frac{\partial f_i}{\partial x}$, i, j = 1,2,3,4 are

continuous.

Considering the partial derivatives;

$$\frac{\partial f_1}{\partial S}\bigg|_{(S_0,I_0,T_0,A_0)} = -\bigg(\frac{w\delta cI}{N} + u\bigg), \\ \left|\frac{\partial f_1}{\partial S}\right| = \bigg|-(\frac{w\delta cI}{N} + u)\bigg| < \infty \\ \text{bounded. Hence (4) has a unique solution.}$$

$$\frac{\partial f_1}{\partial I}\bigg|_{(S_0,I_0,T_0,A_0)} = \bigg|-\frac{w\delta cS}{N}\bigg| = \bigg|\frac{\partial f_1}{\partial I}\bigg| = \bigg|-\frac{w\delta cS}{N}\bigg| < \infty$$
The Basic reproductive number R_0
To calculate the basic reproductive number.

$$\begin{split} \frac{\partial f_1}{\partial T}\bigg|_{(S_0,I_0,T_0,A_0)} &= 0, \left|\frac{\partial f_1}{\partial T}\right| = 0 < \infty \\ \frac{\partial f_1}{\partial A}\bigg|_{(S_0,I_0,T_0,A_0)} &= 0, \left|\frac{\partial f_1}{\partial A}\right| = 0 < \infty \\ \frac{\partial f_2}{\partial S}\bigg|_{(S_0,I_0,T_0,A_0)} &= \left(\frac{w &\&S}{N}\right) = \left|\frac{\partial f_2}{\partial S}\right| = \frac{w &\&S}{N} < \infty \\ \frac{\partial f_2}{\partial I}\bigg|_{(S_0,I_0,T_0,A_0)} &= \frac{w &\&S}{N} - u - \sigma_2 - \sigma_1, \left|\frac{\partial f_2}{\partial I}\right| = \left|\frac{w &\&S}{N} - u - \sigma_2 - \sigma_1\right| < \infty \end{split}$$

$$\frac{\partial f_2}{\partial T}\Big|_{(S_0,I_0,T_0,A_0)} = 0, \frac{\partial f_2}{\partial T}\Big| = 0 < \infty$$

$$\frac{\partial f_2}{\partial A}\Big|_{(S_0,I_0,T_0,A_0)} = 0, \frac{\partial f_2}{\partial A}\Big| = 0 < \infty$$

$$\frac{\partial f_3}{\partial S}\Big|_{(S_0,I_0,T_0,A_0)} = 0, \frac{\partial f_3}{\partial S}\Big| = 0 < \infty$$

$$\frac{\partial f_3}{\partial I}\Big|_{(S_0,I_0,T_0,A_0)} = \sigma_2, \frac{\partial f_3}{\partial I}\Big| = |\sigma_2| < \infty$$

$$\frac{\partial f_3}{\partial T}\Big|_{(S_0,I_0,T_0,A_0)} = u, \frac{\partial f_3}{\partial T}\Big| = |u| < \infty$$

$$\frac{\partial f_3}{\partial A}\Big|_{(S_0,I_0,T_0,A_0)} = v, \frac{\partial f_3}{\partial A}\Big| = |v| < \infty$$

$$\frac{\partial f_4}{\partial S}\Big|_{(S_0,I_0,T_0,A_0)} = 0, \frac{\partial f_4}{\partial S}\Big| = 0 < \infty$$

$$\frac{\partial f_4}{\partial I}\Big|_{(S_0,I_0,T_0,A_0)} = \sigma_1, \frac{\partial f_4}{\partial I}\Big| = |\sigma_1| < \infty$$

$$\frac{\partial f_4}{\partial I}\Big|_{(S_0,I_0,T_0,A_0)} = 0, \frac{\partial f_4}{\partial T}\Big| = 0 < \infty$$

$$\frac{\partial f_4}{\partial I}\Big|_{(S_0,I_0,T_0,A_0)} = 0, \frac{\partial f_4}{\partial I}\Big| = 1$$

Clearly, all $\left| \frac{\partial f_i}{\partial x_i} \right|$, i, j = 1, 2, 3 are continuous and

The Basic reproductive number R_0

To calculate the basic reproductive number (R_0) for the model we adopt Next generation matrix method.

$$G = FV^{-1}$$
 ...8

To calculate the next generation matrix for the SITA model in (1), we need to enumerate the number of ways;

- (1) New infections can arise
- (2)Individuals can move between compartments Though there are two disease states but only one way to create new infections. Hence, we are concerned with I and T compartments of the model. Thus

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$$\frac{dI}{dt} = \frac{w \, \delta c SI}{N} - uI - \sigma_2 I - \sigma_1 I$$

$$\frac{dT}{dt} = \sigma_2 I - uT - vT \qquad ...10$$

...9

From which we obtain

$$F = \begin{pmatrix} \frac{w \delta cs}{N} - (u + \sigma_1 + \sigma_2) & 0\\ 0 & 0 \end{pmatrix}$$
$$V = \begin{pmatrix} (u + \sigma_1 + \sigma_2) & 0\\ -\sigma_2 & (u + v) \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{(u + \sigma_1 + \sigma_2)} & 0 \\ \frac{\sigma_2}{(u + \sigma_1 + \sigma_2)(u + v)} & \frac{1}{(u + v)} \end{pmatrix}$$
 ...11

Therefore

$$G = FV^{-1} = \begin{pmatrix} pqw\delta c & 0 \\ (u + \sigma_2 + \sigma_1)uN & 0 \\ 0 & 0 \end{pmatrix} \dots 12$$

Clearly, It is easy to see that the dominant eigenvalue gives the R_0

$$R_0 = \frac{pqw\delta c}{(u + \sigma_2 + \sigma_1)uN}$$
...13

6.0 Stability Analysis of the model

The equation (1) exhibits two steady states namely: Disease free equilibrium and Endemic Equilibrium.

Disease free equilibrium

$$E_0 = (S^0, I^0, T^0, A^0) = (\frac{pq}{u}, 0, 0, 0)$$
 ...14

and

Endemic Equilibrium.

$$E^* = \left(S^*, I^*, T^*, A^*\right) = \begin{pmatrix} \frac{N(\sigma_2 + u + \sigma_1)}{w \& c}, -\frac{1 - R_0}{uNa^2w\& c}, -\frac{(u\sigma_2 + v\sigma_2 + \alpha\sigma_2 + v\sigma_1)(1 - R_0)}{uNw\&(\sigma_2 + u + \sigma_1)^2(u + \alpha + v)} \\ -\frac{\sigma_1(1 - R_0)}{uNw\&(\sigma_2 + u + \sigma_1)^2(u + \alpha + v)} \end{pmatrix} \dots 15$$

Stability of the Disease Free Equilibrium

To examine the local stability of disease - free equilibrium E_0 we evaluate the Jacobian matrix at $E_0 = (S^0, I^0, T^0, A^0)$.

Considering equation (4) and Let
$${\bf X}=S-S_0$$
 , $I=I$, $T=T$ and $A=A$ The Jacobian matrix is

Preposition: $E_{\scriptscriptstyle 0}$ is local asymptotically stable if

$$[J(E_0) - \lambda I] = \begin{pmatrix} -(u + \lambda) & -\frac{w \delta c S_0}{N} & 0 & 0 \\ 0 & (\frac{w \delta c S_0}{N} - (u + \sigma_2 + \sigma_1)) - \lambda & 0 & 0 \\ 0 & \sigma_2 & -(u + v + \lambda) & v \\ 0 & \sigma_1 & 0 & -(\alpha + u + \lambda) \end{pmatrix}$$

$$\Rightarrow -(u+\lambda)\left[\left(\frac{w\delta cS_0}{N} - (u+\sigma_2+\sigma_1) - \lambda\right)(u+v+\lambda)(\alpha+u+\lambda)\right] = 0 \dots 17$$

Simplifying (17) we have;

$$\lambda_1 = -u \; , \quad \lambda_2 = \frac{w \tilde{\alpha} \cdot S_0}{N} - \left(u + \sigma_2 + \sigma_1\right) \; , \quad \lambda_3 = -(u + v) \; , \quad and \quad \lambda_4 = -\left(v + \alpha + u\right) \quad \dots 18$$

It can be seen from (18) that;

$$R_0 = \frac{pqw\delta c}{(u + \sigma_2 + \sigma_1)uN} < 1 \qquad \dots 19$$

Let
$$R_0 = \frac{w \delta c S_0}{(u + \sigma_2 + \sigma_1)N}$$

Therefore $R_0 < 1$ since all the parameters are non - negative. This proves the proposition.

Stability of Endemic Equilibrium.

To examine the local stability of the endemic equilibrium E_{st} , we evaluated the Jacobian matrix at

$$S_* = \frac{N(\sigma_2 + u + \sigma_1)}{w \delta c} \qquad \dots 20$$

$$I_* = -\frac{1 - R_0}{uNa^2w\delta c} \qquad ...21$$

$$T_{*} = -\frac{(u\sigma_{2} + v\sigma_{2} + \alpha\sigma_{2} + v\sigma_{1})(1 - R_{0})}{uNw\delta c(\sigma_{2} + u + \sigma_{1})^{2}(u + \alpha + v)} \qquad ...22$$

$$A_{*} = -\frac{\sigma_{1}(1 - R_{0})}{uNw\delta c(\sigma_{2} + u + \sigma_{1})^{2}(u + \alpha + v)} \qquad ...23$$

$$A_* = -\frac{\sigma_1(1 - R_0)}{uNw\delta c(\sigma_2 + u + \sigma_1)^2(u + \alpha + v)} \qquad ...23$$

 $S = x + S_*$, $I = y + I_*$, $T = z + T_*$, $A = q + A_*$

The resulting matrix is

$$J(E_*) = \begin{pmatrix} -(\frac{w\delta cI_*}{N} + u) & -\frac{w\delta cS_*}{N} & 0 & 0 \\ \frac{w\delta cI_*}{N} & \frac{w\delta cS_*}{N} - (u + \sigma_2 + \sigma_1) & 0 & 0 \\ 0 & \sigma_2 & -(u + v) & 0 \\ 0 & \sigma_1 & 0 & -(\alpha + u) \end{pmatrix} \dots 24$$

We obtained jacobian matrix as

$$|J(E_*) - \lambda I| = \begin{pmatrix} -(\frac{w \delta c I_*}{N} + u + \lambda) & -\frac{w \delta c S_*}{N} & 0 & 0\\ \frac{w \delta c I_*}{N} & \frac{w \delta c S_*}{N} - (u + \sigma_2 + \sigma_1 + \lambda) & 0 & 0\\ 0 & \sigma_2 & -(u + v + \lambda) & 0\\ 0 & \sigma_1 & 0 & -(\alpha + u + \lambda) \end{pmatrix} \dots 25$$

Solving (25) we obtained,

$$\Rightarrow \lambda^4 + \lambda^3 b_1 + \lambda^2 b_2 + \lambda b_3 + b_4 \qquad \dots 26$$

where,

$$b_1 = \sigma_1 - 2u - v - \alpha - \sigma_2 - \frac{w \delta c S_*}{N} - w \delta c I_*$$
 ...27

$$b_{2} = u\sigma_{1} - u\frac{w\delta cS_{*}}{N} - u\sigma_{2} - u\alpha - uv - u^{2} - \left(\sigma_{1} + \sigma_{2} + \frac{w\delta cS_{*}}{N}\right)(2u + v + \alpha)$$

$$- w\delta cI_{*}\left(u + v + \alpha + \sigma_{2} - \sigma_{1} + \frac{w\delta cS_{*}}{N}\right) + \frac{w\delta cS_{*}}{N}\left[\frac{w\delta cI_{*}}{N}\right]$$
...28

$$b_{3} = u^{3} - u^{2}(\sigma_{2} - v - \alpha - \sigma_{1}) - uv\sigma_{2} - u\alpha\sigma_{2} - uv\sigma_{1} - u\alpha\sigma_{1} - (uv + u\alpha + u^{2})$$

$$+ (2u + uv + u\alpha)\left(\sigma_{1} + \sigma_{2} + \frac{w\delta cS_{*}}{N}\right)\left[1 - w\delta cI_{*}\right] - (2u + v + \alpha)\frac{w\delta cS_{*}}{N}\left[\frac{w\delta cI_{*}}{N}\right] \qquad ...29$$

$$b_{4} = -u^{4} - u(u + v + \alpha) \frac{w \delta c S_{*}}{N} \left[\frac{w \delta c I_{*}}{N} \right] + u^{3} (\sigma_{2} - v - \alpha - \sigma_{1}) + u^{2} (v \sigma_{2} - v \sigma_{1} - \alpha + \alpha \sigma_{1})$$

$$+ (u^{2}v - u^{2} + u^{3}) + u^{3}w \delta c I_{*} (\sigma_{2} - v - \alpha - \sigma_{1})$$

$$+ w \delta c I_{*} \left[uv \sigma_{2} + u \alpha + \sigma_{2} + \sigma_{2}v + u \alpha \sigma_{1} + (uv + u \alpha + u^{2}) \frac{w \delta c S_{*}}{N} \right]$$
...30

Applying Routh - Hurwitz criteria, the endemic equilibrium E_* is local asymptotically stable since $b_1>0,\ b_2>0,\ b_3>0,\ b_4>0,\ b_1b_2-b_3>0$ and $b_2b_2b_3-b_3^2-b_1^2b_4>0$

Numerical simulation of the model.

The numerical simulation of the model is carried out here in order to verify predictions of the model. .

Initial Conditions.

Table 7.1: Assumed Values of Parameters Used for the Data Simulation.

Par	Parameters	Assumed
ame		Values
ters		
Not		
atio		
n		
c	Average number of sexual	1
	partner per unit time	
q	Infant Survival Rate	0.925
p	Proportion of Children that	0.9
	survive after age 15	
β	The contact rate of the	0.5
	epidemic	
\mathcal{E}	The fraction of infected	0.02
	newborn	
γ	Rate of infected newborn baby	0.15
$\sigma_{_1}$	Rate of movement from infected	0.04
	class to AIDS	
σ_2	Rate of movement from infected	0.6
	class to Treatment	
δ	Probability of infection from a	0.4
	sexual contact with infected.	
α	Disease - induced death rate	0.02
W	Sexual contact rate between a	0.5
	sexually mature S person and I	
	person.	

Source: Bashiru K.A (2014)

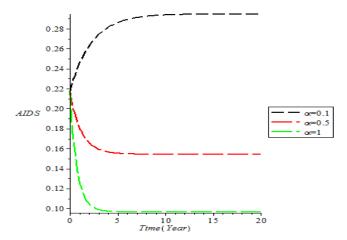


Fig.7.1: Population of AIDS with varying value of lpha

Figure 7.1, shows the effect of disease induced death rate α , it is found that as α increases, the population of AIDS patient's decreases. When $\alpha=0.1$, the AIDS population increased initially and gradually decayed, It is found that the AIDS induced death rate α , can be controlled by providing antiretroviral drugs (ARVs) to AIDS patients or by advising AIDS patients to follow healthy regulation about the AIDS disease. From the figure, it can also be seen that the respective populations are tending to the equilibrium level as time progresses.

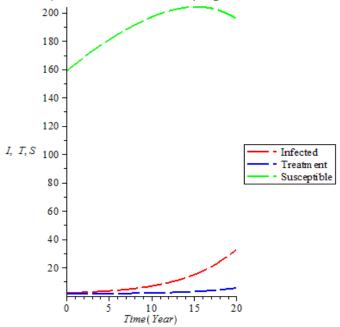


Fig.7.2: Population of Susceptible, Treatment and infected when C=1 and $\delta=0.4, u=0.03, \ w=0.5, \sigma_2=0.02, \sigma_1=0.04$.

Fig 7.2, 7.3 and 7.4 showed the variation of population in Susceptible, Infected and Treated compartments, it was observed that as the sexual contact is increasing the Susceptible population increases and latter started coming down, in fig.7.3 as Sex contact increases the Susceptible decrease, drastically to equilibrium level while the infected and Treated class are increasing due to loss of immunity. But for infective, it can be seen that as C increases the infective also increased. Thus it can be concluded that, in order to reduce the spread of the disease, the number of sexual partners as well as unsafe sexual interaction with an infective should be restricted.

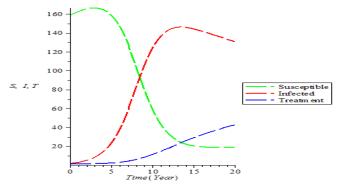


Fig.7.3: Population of Susceptible, Treatment and infected when C=3 and $\delta=0.4, u=0.03, w=0.5, \sigma_2=0.02, \sigma_1=0.04$.

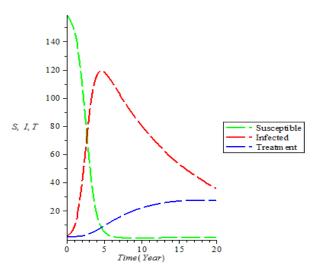


Fig.7.4: Population of Susceptible, Treatment and infected when C=9 and $\delta=0.4, u=0.03, w=0.5, \sigma_2=0.02, \sigma_1=0.04$.

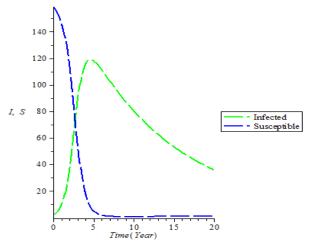
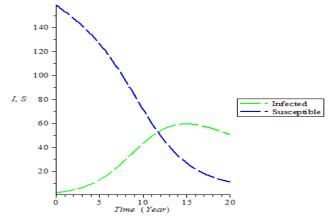


Fig.7.5: Population of Susceptible, infected, when w=0.5, and $\delta=0.4, u=0.03, \ \sigma_2=0.02, \sigma_1=0.04$



$$\delta = 0.4, u = 0.03, \ \sigma_2 = 0.02, \sigma_1 = 0.04.$$

Figure 7.5 and 7.6 predict the Variation of the contact rates of susceptibles with infectives. The figures shows that as the contact rate $\,w\,$ increases the susceptibles decrease, infectives class increased.

Conclusion

In this research, mathematical model of HIV/AIDS was presented compartmentally for Heterosexual mode of transmission. The potential impact of treatment on disease progression was investigated, the analyses showed that an increase in the treatment rate results in an expected decline of new disease incidences.

The basic reproductive number and stability condition of the equilibrium points are derived. From the analysis, it may also be speculated that if the HIV infection is suppressed at an early stage by effectively treating the infectives, the progression to the AIDS can be slowed down and the life span of HIV infectives can be prolonged.

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