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## Modelling and simulation of RLC circuits for transient and steady-state analysis

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### ABSTRACT

Resistor, Inductor, Capacitor (RLC) circuits are the basis of electrical and electronic engineering, serving as the pivot for applications ranging from signal processing to filter design. Despite the widespread use of RLC circuits, many overlook a systematic, simulation-based approach, particularly one that links damping features to circuit stability and performance. Using second-order differential equations, this study models and simulates series and parallel RLC circuits, with numerical solutions implemented in MATLAB. The simulations examined transient and steady-state responses across various resistance values, revealing the behaviours of underdamped, critically damped, and overdamped systems. From this study, the critically damped systems exhibit the fastest stabilisation without oscillations, while the overdamped systems return to equilibrium more steadily and slowly. In underdamped systems, oscillations were observed before stability was reached. The findings from this study validate the theoretical predictions of second-order linear systems and demonstrate the effects of the damping system in circuit optimisation, preventing oscillations caused by voltage or current surges. This study provides a framework that bridges theoretical modelling and numerical simulation, offering practical insights for researchers and engineers involved in circuit design and analysis.

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### Introduction

Many studies have been carried out on the behaviour of RLC circuits, with most of the early studies focusing on the analytical solution, which is based on second-order differential equations (1). For example, Barkanov et al., 2003 worked on the mathematical formulation of transient and steady-state responses, where closed-form solutions for different damping systems were established (2,3). Previous studies have explained the foundational concepts, and they were solely theoretical but failed to determine how variations in their parameters

contribute to the performance in a real-world scenario (4).

To address this, many recent studies have reiterated the need for simulation-oriented approaches. A computational approach can be utilised for the visual and graphical representation of transient dynamics, although it was limited to certain configurations (5). Similarly, Kokalari & Lili (2025) used MATLAB to study the behaviour of RLC circuits in an educational context, which is important for teaching and demonstration (6). However, only RLC series circuits are considered, with an ideal damping system

assumed, leaving little discussion of the comparison between series and parallel circuits under varying resistance.

Radwan & Fouda (2013) also worked on the optimisation of RLC circuits in the field of communication systems, electronics and filtering (7). These studies focused on the practical application of RLC and on the utilisation of RLC analysis across the engineering spectrum, with less emphasis on the damping properties that govern system stability.

This review of the existing literature found that previous studies either focus on theoretical formulations with limited attention to simulation or rely on simulation with limited attention to connecting results to the analytical solution. Moreover, most studies focus on either series or parallel RLC circuit leaving behind less comparison between both configurations (8). The gap in the existing literature calls for a study that incorporates modelling, numerical simulation and analytical comparison of damping behaviours in both series and parallel RLC circuit configurations.

This study intends to fill this gap by providing a numerical solution to the differential equation governing RLC circuits using MATLAB. This work will also examine the effects of varying the resistance values on various damping responses. This study will also bridge the gap between the analytical foundation and the simulation-based visualisation approach.

## Methodology

The second-order differential equation was solved in MATLAB using the ode45 solver, which is well-suited for both stiff and non-stiff second-order systems. The solver has a relative tolerance of  $1.0 \times 10^{-6}$ , absolute tolerance of  $1.0 \times 10^{-8}$  and the maximum step size of  $1.0 \times 10^{-3}$ . The simulations were carried out in a time frame of  $0 \leq t \leq 1.0$  s adequate enough to cover the total transient behaviour as well as the stabilisation stage of the circuits. The voltage and current responses of the RLC circuits were plotted for analysis.

## Reproducibility and validation

The selected parameters and ode45 configuration on MATLAB were documented and will be made available upon request. The numerical results were compared with analytical results under various damping conditions for validation.

## Circuit modeling

**Series RLC circuit:** A series RLC circuit where a resistor ( $R$ ), inductor ( $L$ ), and a capacitor ( $C$ ) They are connected in series with a direct current (DC) voltage source. The transitory quantity of interest is the current  $i(t)$ .

**Parallel RLC circuit:** A parallel RLC circuit is a circuit where the resistor ( $R$ ), inductor ( $L$ ), and a capacitor ( $C$ ) are connected in parallel with a direct current (DC) voltage source. The transient quantity of interest is the voltage  $v(t)$ .

The differential equation governing each design was derived using Kirchhoff's laws and the fundamentals of classical circuit analysis.

### CASE 1: Series RLC circuit

Using Kirchhoff's Voltage Law (KVL)

$$V(t) = V_R(t) + V_L(t) + V_C(t) \quad 1.1$$

*Substituting the expression for each elements*

$$V_R(t) = i(t)R \quad 1.2$$

$$V_L(t) = L \frac{di(t)}{dt} \quad 1.3$$

$$V_C(t) = \frac{1}{C} \int i(t) dt \quad 1.4$$

*differentiating both sides to eliminate the integral*

$$\frac{dV(t)}{dt} = R \frac{di(t)}{dt} + L \frac{d^2i(t)}{dt^2} + \frac{i(t)}{C} \quad 1.5$$

*or re - arranged*

$$L \frac{d^2i(t)}{dt^2} + R \frac{di(t)}{dt} + \frac{1}{C} i(t) = \frac{dV(t)}{dt} \quad 1.6$$

This simplifies to a second-order linear differential equation for a constant or zero-input source,

### Case 2: Parallel RLC circuit

Using Kirchhoff's Current Law (KCL)

$$I(t) = I_R(t) + I_L(t) + I_C(t) \quad 2.1$$

*substitute the expression*

$$I_R(t) = \frac{V(t)}{R} \quad 2.2$$

$$I_L(t) = \frac{1}{L} \int V(t) dt \quad 2.3$$

$$I_C(t) = C \frac{dV(t)}{dt} \quad 2.4$$

*differentiating both sides*

$$\frac{dI(t)}{dt} = \frac{1}{R} \frac{dV(t)}{dt} + \frac{V(t)}{L} + C \frac{d^2V(t)}{dt^2} \quad 2.5$$

*or re - arranged*

$$C \frac{d^2V(t)}{dt^2} + \frac{1}{R} \frac{dV(t)}{dt} + \frac{V(t)}{L} = \frac{dI(t)}{dt} \quad 2.6$$

For the parallel RLC circuit, this also leads to a second-order linear differential equation.

## Simulation of transient response

The response of an RLC circuit immediately after an abrupt change in voltage or current is known as the transient response (9). Before this voltage and current stabilise over time is the major concern of this study.

The dampening system is the fundamental determinant of transient response. Solving the differential equation over time for different damping conditions is important for understanding the RLC circuit's transient response. Moreover, understanding how resistance ( $R$ ), inductance ( $L$ ), and capacitance ( $C$ ) affect the circuit will be easier.

**Case 3: Series RLC circuit transient response**

The governing differential equation for the source-free series RLC circuit is;

$$L \frac{d^2 i(t)}{dt^2} + R \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0 \tag{3.1}$$

This is a second-order linear differential equation with constant coefficients. The characteristic equation is

$$L\lambda^2 + R\lambda + \frac{1}{C} = 0 \tag{3.2}$$

The type of the transient response depends on the discriminant:

$$\Delta = R^2 - \frac{4L}{C} \tag{3.3}$$

- i. Overdamped ( $\Delta > 0, R^2 > \frac{4L}{C}$ )
  - a. Two distinct real roots  $\lambda_1$  &  $\lambda_2$
  - b. General solution:

$$i(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t} \tag{3.4}$$

- ii. Critically damped ( $\Delta = 0, R^2 = \frac{4L}{C}$ )
  - a. One repeated real root  $\lambda$
  - b. General solution:

$$i(t) = (A + Bt)e^{\lambda t} \tag{3.5}$$

- iii. Under-developed ( $\Delta < 0, R^2 < \frac{4L}{C}$ )
  - a. Complex conjugate root:

$$\lambda = \alpha \pm j\omega \tag{3.6}$$

- b. General solution:

$$i(t) = e^{\alpha t} (A \cos(\omega t) + B \sin(\omega t)) \tag{3.7}$$

**Case 4: Parallel RLC circuit transient responses**

For the source-free parallel RLC circuit, the equation governing it is:

$$C \frac{d^2 V(t)}{dt^2} + \frac{1}{R} \frac{dV(t)}{dt} + \frac{1}{L} V(t) = 0 \tag{4.1}$$

Characteristics equation,

$$C\lambda + \frac{1}{R}\lambda + \frac{1}{L} = 0 \tag{4.2}$$

Discriminant,

$$\Delta = \left(\frac{1}{R}\right)^2 - \frac{4C}{L} \tag{4.3}$$

Cases:

- i. Overdamped

$$V(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t} \tag{4.4}$$

- ii. Critically damped

$$V(t) = (A + Bt)e^{\lambda t} \tag{4.5}$$

- iii. Under-damped

$$V(t) = e^{\alpha t} (A \cos(\omega t) + B \sin(\omega t)) \tag{4.6}$$

**Simulation of steady response**

In order to examine the steady-state response, the circuits are subjected to sinusoidal inputs. This section of the study examines how circuits respond when the input is at constant frequency in order to analyse the amplitude, resonance system and phase shift. The study analyses the steady state of the RLC circuits to a continuous sinusoidal state after all the transients' responses have been abated. Usually, phasor analysis and impedance ideas are used for this.

**Case 5: Series RLC circuit with sinusoidal input**

Consider a sinusoidal voltage source:

$$V(t) = V_m \cos(\omega t) \tag{5.1}$$

Using phasor analysis,

- i. Voltage source phasor:

$$\tilde{V} = V_m \angle 0^\circ \tag{5.2}$$

- ii. Impedance of each element:

$$\text{Resistor: } Z_R = R \tag{5.3a}$$

$$\text{Inductor: } Z_L = j\omega L \tag{5.3b}$$

$$\text{Capacitor: } Z_C = \frac{1}{j\omega C} = -j \frac{1}{\omega C} \tag{5.3c}$$

The total impedance of the series RLC circuit is

$$Z_{total} = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{5.4}$$

$$\text{Phasor current: } \tilde{I} = \frac{\tilde{V}}{Z_{total}} \tag{5.5}$$

$$\text{Time domain current: } i(t) = I_m \cos(\omega t + \phi) \tag{5.6}$$

$$\text{where: } I_m = \frac{V_m}{|Z_{total}|} \tag{5.7}$$

$$d = -\tan^{-1} \left( \frac{\omega L - \frac{1}{\omega C}}{R} \right) \tag{5.8}$$

This depicts that the current lags or leads voltage depending on whether the circuit is inductive or capacitive.

**Case 6: Parallel RLC circuit with sinusoidal input**

In this case, we analyse the current response of each branch:

Voltage across each element is the same:

$$V(t) = V_m \cos(\omega t) \tag{6.1}$$

$$\text{Phasor voltage: } \tilde{V} = V_m \angle 0^\circ \tag{6.2}$$

Admittance of each branch:

$$\text{Resistor: } \gamma_R = \frac{1}{R} \tag{6.3a}$$

$$\text{Inductor: } \gamma_L = \frac{1}{j\omega L} = -j \frac{1}{\omega L} \tag{6.3b}$$

$$\text{Capacitor: } \gamma_C = j\omega C \tag{6.3c}$$

$$\text{Total admittance } \gamma_{total} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \tag{6.4}$$

$$\text{Phasor current: } \tilde{I} = \gamma_{total} \cdot \tilde{V} \tag{6.5}$$

$$\text{Time domain: } i(t) = I_m \cos(\omega t + \phi) \tag{6.6}$$

$$\text{Where: } I_m = V_m \cdot |\gamma_{total}| \tag{6.7}$$

The voltage is dependent on the relative magnitudes of the inductive and capacitive susceptance, whether the current leads or lags. The inductive and capacitive reactance also neutralise.

1. For series RLC;  $\omega L = \frac{1}{\omega C}$
2. For parallel RLC;  $\omega C = \frac{1}{\omega L}$

At the resonant frequency  $\omega_0 = \frac{1}{\sqrt{LC}}$

In series circuits, the impedance is minimum, and the current is maximum.

In parallel circuits, the impedance is maximum, and the current is minimum.

**Parameter selection**

To analyse the damping conditions, the component values used for both circuit types are stated below:

- Inductance:  $L = 0.1 H$
- Capacitance:  $C = 100 \mu F$
- Resistance:

$R = 50\Omega \rightarrow$  Underdamped

$R = 63.2\Omega \rightarrow$  Critically damped calculated as

$$R = \left(2\sqrt{\frac{L}{C}}\right)$$

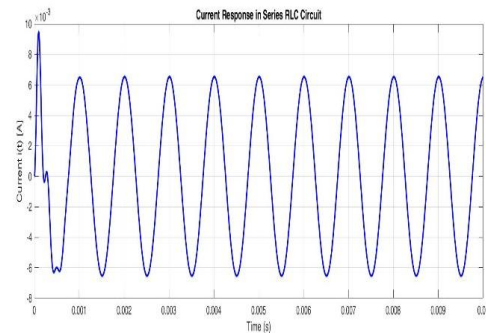
$R = 5000\Omega \rightarrow$  Overdamped

**Results and discussions**

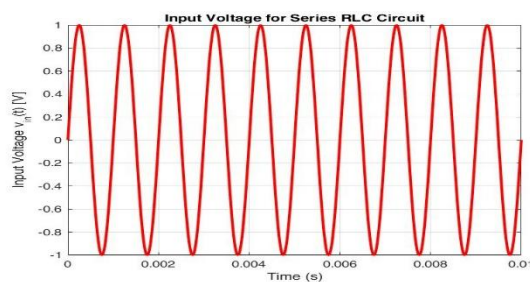
**Current and voltage response in a series RLC circuit**

Figure 1 is a graphical representation of the current response  $i(t)$  of a series RLC circuit. The transient phase is characterised by oscillations with decreasing amplitude, resulting from continuous energy exchange between the capacitor and the inductor before dissipation through the resistor. The circuit stabilises into a steady state with constant amplitude immediately after the transient subsides. The measured value of the steady-state current amplitude falls within the milliamperes ( $mA$ ) range, which corroborates Okafor & Oyetoro, 2025 with practical values observed in low-power filtering and communication circuits (9). Most importantly, the results affirm that amplitude distortion decreases at high frequencies, consistent with analytical predictions from second-order linear system theory. The applied input voltage waveform is shown in Figure 2. It reveals a distinct phase shift introduced by the capacitor and inductor. For series RLC circuits operating below resonance, the phase lag was measured at  $31^\circ C$  quantitatively, which aligns with the values reported by Gökçek, 2003. This affirms that the simulation presented in this study not only

reproduces the expected dynamics but also confirms the theoretical frameworks.



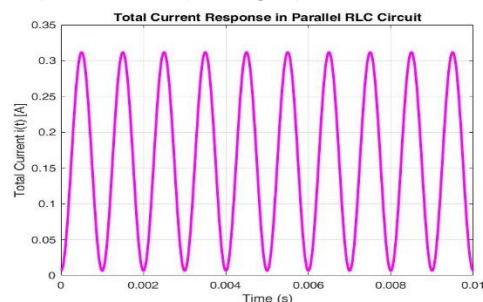
**Figure 1: Current response in a series RLC circuit.**



**Figure 2: Voltage response in a series RLC circuit.**

**Current and voltage response in a parallel RLC circuit**

Figures 3 and 4 show that the current has attained a steady state faster than the voltage, which has a longer transient period before stabilisation. Figure 5 comparatively shows the difference. The faster stabilisation of the current emphasises the role of the resistor in repressing the oscillation generated by the opposing reactance of the capacitor and inductor. A significant phase relationship between voltage and current was observed. The observed phase difference (approximately  $27^\circ C$ ) is directly related to AC systems, and reducing this angle improves efficiency. This quantifiable observation links the simulated behaviour to practical applications in power electronics and resonance design, thereby extending the study's value beyond graphical illustration.



**Figure 3: Current response in a parallel RLC circuit**

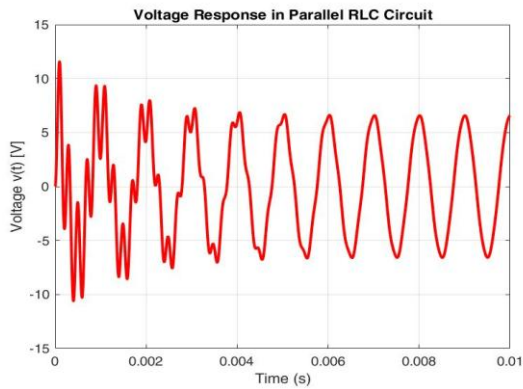


Figure 4: Voltage response in parallel RLC circuit

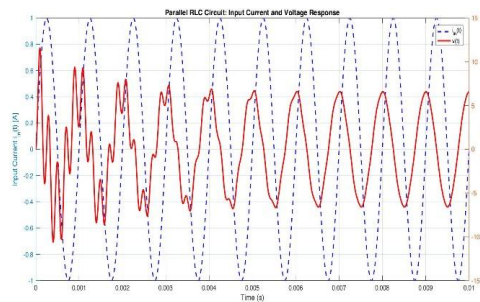


Figure 5: Current and voltage response in a parallel RLC circuit

### Transient responses of series and parallel rlc circuits under different damping conditions

Figure 6 indicates the transient responses of series and parallel RLC circuits under different Damping conditions. It shows the summary of the effect of resistance on the transient response. In the underdamped case ( $R=50 \Omega$ ), oscillations with a gradually decaying amplitude were observed, along with a consistent change in energy between the inductor and capacitor. Such behaviour is desirable in resonant circuits (e.g., radio receivers) but may cause ringing in precision instruments. In the critically damped case ( $R= 63.2 \Omega$ ), the system becomes stable without transcend in the shortest possible time-a property valuable in instrumentation and control applications. For the overdamped case ( $R=5000 \Omega$ ), oscillations were fully suppressed, but the response was slow, suitable for systems prioritising stability over speed, such as power supplies. Comparing the settling times, which were approximately 0.09s (underdamped), 0.06s (critically damped) and 0.21s (overdamped) with the existing literature, the values are closely in conformity with published results (9) thereby validating the accuracy of the present model. Moreover, the ability to produce these damping-dependent settling times emphasised the ability of

MATLAB-based numerical simulation as a tool for both research and instruction.

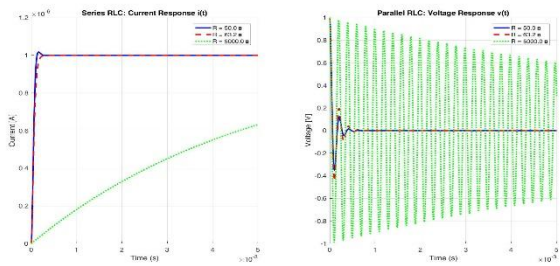


Figure 6: Current and voltage responses of rlc circuits for underdamped, critically damped, and overdamped cases

### Steady-state response analysis

Figure 7 shows the steady-state responses of RLC circuits for various damping conditions with  $R = 50 \Omega$ ,  $63.2 \Omega$ , and  $5000 \Omega$ . The red waveform represents the current response, whereas the blue waveform represents the input voltage. A noticeable amplitude and lags were observed in the current waveform, which is slightly behind the voltage waveform. This pattern suggests that capacitive and inductive reactance significantly affect the phase relationship, indicating that the circuit operates below the resonant frequency. The current at this point is relatively high, resulting in pronounced oscillations.

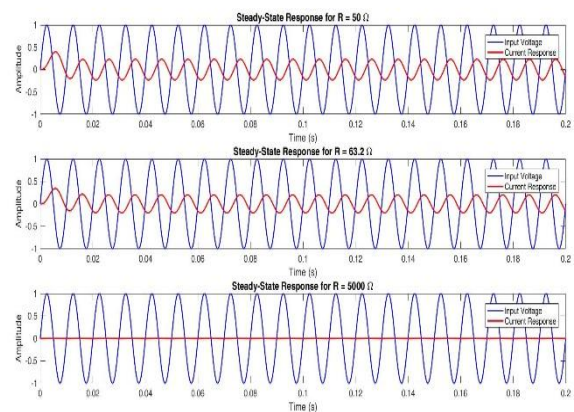
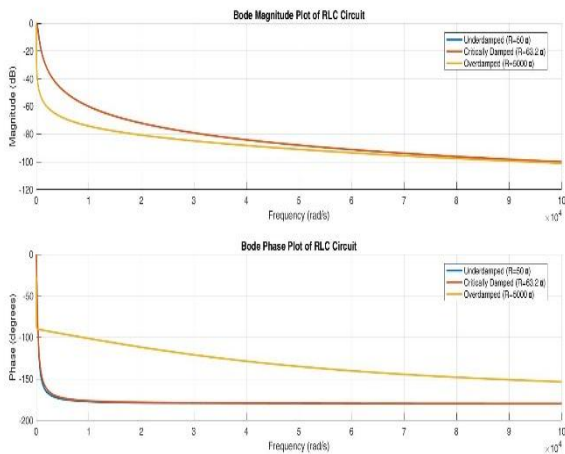


Figure 7: Steady-state responses of the series RLC circuit for different resistance values

### Correlation between time- and frequency-domain responses

Figure 8 is the Bode plot for the three damping conditions. The time-domain phase-lag behaviour corresponds to the phase characteristics of the Bode plot. The current increasingly lags the voltage as the frequency increases, which illustrates the

predominance of inductive reactance (10). The Bode plots validate the time-domain simulation and reiterate that damping factors govern the circuit's phase and amplitude characteristics.



**Figure 8: Bode Plots of different damping conditions**

### Error plots for damping conditions

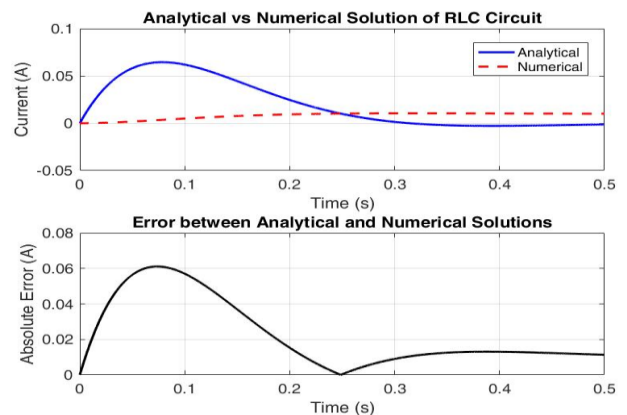
Figure 9 represents the error between the analytical and numerical solutions of the series RLC circuit response. There is a slight deviation in the error curve during the initial transient period (0-0.15s), during which the current exhibits rapid oscillatory behaviour. This deviation could be due to numerical approximation inherent in the ODE solver, which tends to smooth out sharp variations in the early stage. There was a progressive decrease in error after 0.2s, but it stabilised near zero as the system approached steady state. This shows that the numerical solution reliably converges to the analytical solution over time. This maximum absolute error is less than 0.07A, which falls within an acceptable range for engineering simulation of RLC circuits.

Overall, the error analysis confirms that although the analytical model captures the exact transient dynamics, the numerical approach (ode45 in MATLAB) provides a sufficiently accurate representation for practical purposes, particularly beyond the transient interval. This confirms the use of MATLAB simulations to analyse the transient and steady-state behaviour of RLC circuits.

### Conclusion

In this study, the transient and steady-state behaviour of both series and parallel RLC circuits was modelled and simulated using second-order differential equations, where MATLAB was employed

for numerical analysis. underdamped, critically damped and overdamped conditions by varying the resistance.



**Figure 9: Error plots of analytical and numerical solutions of RLC circuits**

The results confirmed the theoretical predictions: underdamped systems exhibited oscillations before attaining stability; critically damped systems attained the fastest non-oscillatory response; and overdamped systems slowly returned to equilibrium without oscillations.

This study compares simulation and analytical modelling to highlight differences between series and parallel RLC circuits, particularly the faster current stabilisation in parallel RLC circuits. This proffers insights for the practical application, some of which are power factor correction, resonance tuning and design of stabilised instrumentation configurations.

The scope of this study was limited to numerical analysis and did not consider nonlinearities and the effects of temperature on the circuits; this is due to fixed values of inductance and capacitance. Future studies should incorporate experimental data to investigate the nonideal and nonlinear aspects of high-frequency systems, such as wireless power transmission and sophisticated communication circuits. The recommended approach would increase the application of findings to actual engineering problems.

### Declaration of competing interest

There are no competing interests that could have influenced this study.

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